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MINIMAL RATIONAL THREEFOLDS II

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The Enriques-Fano classification ([E.F], [F]) of the maximal connected algebraic subgroups of the three variable Cremona group, despite of its group theoretic feature, seems to be the most significant result on the rational threefolds so far known. In this paper as in [MU] we interpret the Enriques-Fano classification from a geometric view point, namely the geometry of minimal rational threefolds. We explained in [MU] the link between the two objects; the maximal algebraic subgroups and the minimal rational threefolds. Let (G, X) be a maximal algebraic subgroup of three variable Cremona group. We denote by $\mathscr{C}(G, X)$ the set of all the algebraic operations (G, Y) such that Y is non-singular and projective and such that (G, Y) is isomorphic to (G, X) as law chunks of algebraic operation: namely (G, Y) is birationally equivalent to (G, X). Then we define an order in $\mathscr{C}(G, X)$: for (G, Z), $(G, W) \in \mathscr{C}(G, X)$, $(G, Z) > (G, W)$ if there exists an G-equivariant birational morphism of Z onto W.

Using the classification of [U4], we can state our result.

If (G, X) *is one of the maximal algebraic subgroups except for* (J9) *and* (J11) *listed in Theorem* (2.1), [U4], *then there exists the unique minimal element in the ordered set* $\mathscr{C}(G, X)$ *and any other element of* $\mathscr{C}(G, X)$ *is an equivariant blow-up of the minimal element. For the operations* (J9) *and* (J11), *we can describe the relatively minimal elements in* $\mathscr{C}(G, X)$; *there are countable many relatively minimal elements and they are explicitly constructed and are related each other by the equivariant elementary transformation. In these cases too, any other element is an equivariant blow-up of a relatively minimal elements.*

Since our result thus reveals a new fascinating corner where the simplicity dominates, we have not tried to relate our result with the recent attempts of constructing minimal models for threefolds allowing

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