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MINIMAL LINKAGE AND THE GORENSTEIN LOCUS OF AN IDEAL

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Introduction

Let I be a Cohen-Macaulay ideal of grade $g > 0$ in a local Gorenstein ring (R, m) with residue class field k. An R-ideal J is said to be linked to I with respect to the regular sequence $\underline{\alpha} = \alpha_1, \cdots, \alpha_g \subset I \cap J$ if $J = (\underline{\alpha}) : I$ and $I = (\underline{\alpha}) : J$ ([6]). In this paper we are concerned with the following question: how big is $\dim_k((\underline{\alpha}, mJ)/mJ)$? Obviously this dimension is at most g, but it could be as small as 0. If it is g then the link from J to I is called a minimal link, which is in most respects the desired type of link. The only general result known in this direction is that if I is Gorenstein, then $\dim_k((\underline{\alpha}, mJ)/mJ) = g$ unless both I and J are complete intersections (see [1], Proposition 5.2). We are able to generalize this fact to the case where $(R/I)_p$ is Gorenstein for all prime ideals p in R/I with $\dim(R/I)_p \leq 4$; however we have to assume that I is generically a complete intersection ideal, and that R is a complete intersection (Theorem 2.3). Without the assumption on R we prove that if I is generically a complete intersection, and if for a fixed integer r the type of $(R/I)_p$ is at most r for all prime ideals p in R/I with $\dim(R/I)_p \leq (r+1)^2$, then $\dim_k((\underline{\alpha}, mJ/mJ)) \geq g - r$ (Proposition 2.1). If $r = 1$, i.e. if R/I is Gorenstein in codimension 4, then this estimate shows the dimension is at least $g - 1$. Theorem 2.3 can also be interpreted to yield a strong upper bound for the codimension of the non-Gorenstein-locus of certain perfect ideals: Let R be a regular local ring. Let I be an R-ideal which is generically a complete intersection, and assume that I is in the even linkage class of a Gorenstein ideal (i.e., there exists a sequence of links $I \sim I_1 \sim I_2 \sim \cdots \sim I_{2n}$ with I_{2n} a Gorenstein ideal); then I is a Gorenstein ideal provided that $(R/I)_p$ is Gorenstein for all prime ideals p of R/I with $\dim(R/I)_p \leq 4$ (Corollary 3.1).

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