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ON THE JACOBIAN EQUATION $J(f,g)=0$ FOR POLYNOMIALS IN $k[x,y]$

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Let $k[x, y]$ be the ring of polynomials in two variables over a field k of characteristic zero.

If $f, g \in k[x, y]$ then we write $f \sim g$ in the case where $f = ag$, for some $a \in k^* = k \setminus \{0\}$, and we denote by $[f, g]$ the jacobian of (f, g), that is, $[f, g] = f_x g_y - f_y g_x$.

By a *direction* we mean a pair (p, q) of integers such that $\gcd(p, q) = 1$ and $p > 0$ or $q > 0$. If (p, q) is a direction then we say that a non-zero polynomial $f \in k[x, y]$ is a *(p, q)-form of degree n* if f is of the form

$$f = \sum_{pi+qj=n} a_{ij} x^i y^j ,$$

where $a_{ij} \in k$.

The following two facts are well known

THEOREM 0.1 ([1], [3], [2]). *Let (p, q) be a direction and let f and g be (p, q)-forms of positive degrees. If $[f, g] = 0$ then there exists a (p, q)-form h such that $f \sim h^m$ and $g \sim h^n$, for some natural m, n.*

THEOREM 0.2 ([2], [7]). *Let f and g be polynomials in $k[x, y]$ and assume that $[f, g]$ is a non-zero constant. Put $\deg(f) = dm > 1$, $\deg(g) = dn > 1$, where $\gcd(m, n) = 1$. Let W_f and W_g be the Newton's polygons of f and g, respectively. Then the polygons W_f and W_g are similar. More precisely, there exists a convex polygon W with vertices in $Z \times Z$ such that $W_f = mW$ and $W_g = nW$.*

Theorem 0.1 plays an essential role in considerations about the Jacobian Conjecture (see for example [1], [3], [2], [5]). Theorem 0.2 is also a consequence of Theorem 0.1.

In this note we show that Theorem 0.1 is a special case of a more general fact. We prove (see Section 1) that if f and g are non-constant

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