R. Sasaki
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GENERALIZED ONO INVARIANT AND RABINOVITCH'S THEOREM FOR REAL QUADRATIC FIELDS

RYUJI SASAKI

§ 1. Introduction

Let d be a square-free integer. Let

$$\omega = \begin{cases} \sqrt{d} & \text{if } d \equiv 2,\ 3 \pmod 4 \\ \frac{1}{2}(1+\sqrt{d}) & \text{if } d \equiv 1 \pmod 4, \end{cases}$$

and $\{1, \omega\}$ forms a Z-basis for the ring of integers of the quadratic field $\boldsymbol{Q}(\sqrt{d})$. We denote by Δ and h_d the discriminant and the class number of $\boldsymbol{Q}(\sqrt{d})$, respectively. We define the polynomial $P(X)$ by

$$P(X) = X^2 + \mathrm{Tr}(\omega)X + \mathrm{Nm}(\omega)$$

where Tr and Nm are the trace and the norm. When d is negative, i.e., $\boldsymbol{Q}(\sqrt{d})$ is an imaginary quadratic field, T. Ono define the natural number p_d by

$$p_d = \operatorname*{Max}_{0 \leqq a \leqq \frac{1}{4}|\Delta|-1} \deg P(a) \qquad d \neq -1,\ -3,$$
$$p_{-1} = p_{-3} = 1.$$

Here, for a positive integer N, $\deg N$ means the number of prime divisors of N (counting multiplicity). Concerning the Ono invariant p_d, we have the following ([8], [9]):

THEOREM. *Assume $d < 0$, then we have*

(1) $$p_d \leqq h_d,$$

(2) $$p_d = 1 \Longleftrightarrow h_d = 1,$$

(3) $$p_d = 2 \Longleftrightarrow h_d = 2.$$

(2) is so-called Rabinovitch's theorem. In this paper we define p_d

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