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ON BRAUER'S HEIGHT 0 CONJECTURE

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R. Brauer not only laid the foundations of modular representation theory of finite groups, he also raised a number of questions and made conjectures (see [1], [2] for instance) which since then have attracted the interest of many people working in the field and continue to guide the research efforts to a good extent. One of these is known as the "Height zero conjecture". It may be stated as follows:

CONJECTURE. *Let B be a p-block of the finite group G. All irreducible ordinary characters of G belonging to B are of height* 0 *if and only if a defect group of B is abelian.*

The conjecture is known to be true in special cases: Reynolds [14] treated the case of a normal defect group. Fong [6] proved the "if"-part for p-solvable groups and the "only if"-part for the principal block of a p-solvable group. Very recently, the proof for p-solvable groups has been completed by papers of Wolf and of Gluck and Wolf jointly [15], [8], [9].

The present paper deals with the "if"-part of the conjecture. We show that this part holds true provided it holds for all quasi-simple groups, i.e. for the covering groups of non-abelian simple groups and their factor groups.

Since the finite simple groups are classified, there is good reason to assume that in due time, the conjecture will be verified for, or a counter-example will show up among, these groups. In fact, this should be just a byproduct of a general effort to study the representation theory of simple groups. Relevant results are contained in papers of Fong-Srinivasan [7] and Michler-Olsson [13].

We should mention that, ironically, this way of proving the "if"-part of Brauer's conjecture runs contrary to Brauer's intentions. He seems to have hoped to somehow structure the theory of simple groups using

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