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CATEGORIES OF MULTIPLICATIVE FUNCTORS AND WEIL'S INFINITELY NEAR POINTS

Dedicated to C. Ehresmann in the commemoration of his 80th birthday

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§ 1. Introduction

At the early fifties A. Weil introduced [3] and algebraic approach to the theory of infinitesimal prolongations of smooth manifolds motivated by the theory of jets developed by Ch. Ehresmann on one side and also by the return to Fermat's methods on the infinitesimal calculus of first order, that makes use of nilpotent infinitesimals.

We start working with associative, commutative, unitary, finite dimensional $\boldsymbol{R}$-algebras A having a nilpotent ideal I complementary to $\boldsymbol{R}$, this identified with its image by the map $\boldsymbol{R} \to A$ $t \to t.1_A$. In this case $A = \boldsymbol{R} \oplus I$ and I is the unique maximal ideal of A. Following Weil we will call these algebras local algebras. The morphisms of $\boldsymbol{R}$-algebras will be the usual ones, that is, $\boldsymbol{R}$-linear and compatible with the subjacent ring structure.

The manifolds will be supposed C^∞, Hausdorff, second countable, with no restriction on the dimension of components, unless explicited stated, all maps between manifolds C^∞; this category of objects and morphisms will be denoted by $\mathfrak{M}$. Then in Weil's sense an A-near point over a manifold M is a morphism of $\boldsymbol{R}$-algebras $C^\infty(M) \xrightarrow{\eta} A$. It is a basic result that there is a canonical one-to-one correspondence between $\boldsymbol{R}$-near points over M and the points of M itself, namely $x \mapsto \varepsilon_x$, where ε_x is the evaluation morphism $C^\infty(M) \to \boldsymbol{R}$, $f \mapsto f(x)$. An A-near point η over M is said to be (infinitely) near to the point x of M if (and only if) $p \circ \eta$ corresponds to x by the bijection described above; here $A \xrightarrow{p} \boldsymbol{R}$ is the canonical projection. The Ehresmann's q^k-velocities over M originates in this point of view from the k-truncated polynomial algebra in q indeterminates with real coefficients.

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