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LITTLEWOOD-PALEY THEORY ON GAUSSIAN SPACES

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§ 1. Introduction

In this article we prove a number of inequalities of Littlewood-Paley-Stein (LPS) type for functions on general Gaussian spaces (s. below).

In finite dimensional Euclidean spaces (with Lebesgue measure) the power of such inequalities has been demonstrated in Stein's book [12]. In his second book [13], Stein treats other spaces too: also the situation of a general measure space (X, μ). However the latter case is too general to allow for a rich class of inequalities (cf. Theorem 10 in [13]).

Meyer has proved in [6] a large set of LPS inequalities for the Wiener space, i.e. larger than in Stein's theorem quoted above. This is of course possible due to the concrete nature of the Wiener measure. As an application, Meyer has proved in [7] an equivalence of certain norms on Wiener space and the algebraic structure of a space of "smooth" Wiener functionals. These results have in turn important applications in the Malliavin calculus. The generalizations of these results for general Gaussian spaces will be discussed in a forthcoming paper.

Here we consider the following situation: let $\mathcal{N}$ be a real, separable, nuclear pre-Hilbert space with scalar product $(\cdot, \cdot)$, compatible with the nuclear topology of $\mathcal{N}$. $\mathcal{H}$ denotes the completion of $\mathcal{N}$ in the norm $|\cdot|$ induced by $(\cdot, \cdot)$, and $\mathcal{N}^*$ is the (topological) dual of $\mathcal{N}$. By $\mathcal{B}$ we denote the σ-algebra generated by the cylinder sets of $\mathcal{N}^*$ and $d\mu$ is the Gaussian measure on $\mathcal{N}$ defined by the scalar product on $\mathcal{H}$ (via the Bochner-Minlos theorem [3, 5]):

$$\int_{\mathcal{N}^*} \exp(i\langle x, \xi\rangle) d\mu(x) = \exp\left(-\frac{1}{2}|\xi|^2\right) \tag{1.1}$$

for $\xi \in \mathcal{N}$, $\langle\cdot, \cdot\rangle$ denoting the pairing of $\mathcal{N}^*$ and $\mathcal{N}$. We refer to the triple $(\mathcal{N}^*, \mathcal{B}, d\mu)$ as a *Gaussian space.*

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