Y. Odai
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ON UNRAMIFIED CYCLIC EXTENSIONS OF DEGREE l OF ALGEBRAIC NUMBER FIELDS OF DEGREE l

YOSHITAKA ODAI

Introduction

Let l be an odd prime number and let K be an algebraic number field of degree l. Let M denote the genus field of K, i.e., the maximal extension of K which is a composite of an absolute abelian number field with K and is unramified at all the finite primes of K. In [4] Ishida has explicitly constructed M. Therefore it is of some interest to investigate unramified cyclic extensions of K of degree l, which are not contained in M. In the preceding paper [6] we have obtained some results about this problem in the case that K is a pure cubic field. The purpose of this paper is to extend those results.

Let $\boldsymbol{Q}$ denote the field of rational numbers and let Z be the ring of rational integers. Let ζ be a primitive l-th root of unity. Let $k = \boldsymbol{Q}(\zeta)$ and $L = K(\zeta)$. In Section 1 we see how an unramified cyclic extension N of K of degree l is obtained from an element α of L. Here α satisfies some conditions, one of which is that there exists an ideal $\mathfrak{A}$ of L such that $(\alpha) = \mathfrak{A}^l$. In Section 2, assuming that L is a ramified Galois extension of k, we give a criterion for N to be contained in M by means of α (see Theorem 1). In Section 3, assuming that l is regular, we define F_1 (resp. F_0) as the composite of all those N, for which $\mathfrak{A}$ are ambigious over k (resp. principal) (see Definition). Theorem 2 proves that $F_1 = F_0M$. In Section 4 F_0 is investigated and Theorem 4 gives infinitely many examples of N not contained in M.

NOTATIONS. $G = \mathrm{Gal}(L/K)$ is a cyclic group of order $l-1$. Let τ be a generator of G and let $\dot{r}$ be the element of Z/lZ such that $\zeta^\tau = \zeta^{\dot{r}}$. Let $Z/lZ[G]$ denote the group ring of G over Z/lZ. We define

$$\dot{e}_i = -\sum_{j=0}^{l-2} \dot{r}^{-ij}\tau^j \qquad \text{for } 1 \leqq i \leqq l-1 .$$

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