Y. Miyata
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VERTICES OF IDEALS OF A $\mathfrak{p}$-ADIC NUMBER FIELD II

YOSHIMASA MIYATA

Let k be a $\mathfrak{p}$-adic number field with the ring $\mathfrak{o}$ of all integers in k, and K be a finite normal extension with Galois group G. Π denotes a prime element of the ring $\mathfrak{O}$ of all integers in K. Then, an ideal (Π^a) of $\mathfrak{O}$ is an $\mathfrak{o}G$-module. E. Noether [5] showed that if K/k is tamely ramified, $\mathfrak{O}$ is a free $\mathfrak{o}G$-module. A. Fröhlich [2] generalized E. Noether's theorem as follows: $\mathfrak{O}$ is relatively projective with respect to a subgroup S of G if and only if $S \supseteqq G_1$, where G_1 is the first ramification group of K/k. Now we define the vertex $V(\Pi^a)$ of (Π^a) as the minimal normal subgroup S of G such that (Π^a) is relatively projective with respect to a subgroup S of G (cf. [7] § 1). Then, the above generalization by A. Fröhlich implies $V(\mathfrak{O}) = G_1$. In the previous paper [7], we proved $G_1 \supseteqq V(\Pi^a) \supseteqq G_2$, where G_2 is the second ramification group of K/k (cf. [7] Theorem 5). Further, we dealt with the case where $G = G_1$ is of order p^2, and proved that if $V(\Pi^a) \supsetneqq G_1$, then $a \equiv 1 (p^2)$ and $t_2 \equiv 1 (p^2)$ for the second ramification number t_2 of K/k (cf. [7] Theorems 15 and 21). The purpose of this paper is to prove the similar theorem for the wildly ramified p-extension of degree p^n (Theorem 7).

Throughout this paper, we assume that p is an odd prime and the p-extension K/k is wildly ramified. In the first section § 1, we shall prove that (Π^a) is an indecomposable $\mathfrak{o}G$-module under the assumption relating to the ramification numbers of subextension of K/k (Theorem 2), which is a generalization of S.V. Vostokov's theorem concerning to the indecomposablity of ideals (Π^a) of abelian p-extensions ([10] Theorem 5). In the second section § 2, we shall deal with the case where G_2 is of order p, and we shall prove that if $a \not\equiv 1 \ (|G_1|)$, then $V(\Pi^a) = G_1$, where $|G_1|$ denotes the order of G_1 (Theorem 6). In the last section § 3, we shall prove that if $V(\Pi^a) \supsetneqq G_1$ and $t_1 = 1$, then $a \equiv 1 \ (|G_1|)$ and $t_i \equiv 1 \ (|G_1/G_{i+1}|)$ for $1 \leqq i \leqq r$, where $t_1, t_2, \cdots, t_r$ are ramification numbers of K/k and G_i is

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