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CERTAIN UNITARY REPRESENTATIONS OF THE INFINITE SYMMETRIC GROUP, II

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Introduction

The *infinite symmetric group* $\mathfrak{S}_\infty$ is the discrete group of all finite permutations of the set X of all natural numbers. Among discrete groups, it has distinctive features from the viewpoint of representation theory and harmonic analysis. First, it is one of the most typical ICC-groups as well as free groups and known to be a group of non-type I. Secondly, it is a locally finite group, namely, the inductive limit of usual symmetric groups $\mathfrak{S}_n$. Furthermore it is contained in infinite dimensional classical groups $GL(\infty)$, $O(\infty)$ and $U(\infty)$ and their representation theories are related each other.

Our present interest lies in irreducible unitary representations of $\mathfrak{S}_\infty$. Its factor representations of type II have been studied considerably in [6]. While, its irreducible representations have been investigated only in a few particular cases, see [1] and [4]. So it is important to have a large stock of irreducible representations. The present paper is a continuation of the author's previous one [3], where we have discussed irreducible representations of $\mathfrak{S}_\infty$ parametrized by certain automorphisms of X.

Let $\mathrm{Aut}(X)$ be the group of all automorphisms of X. For each $\theta \in \mathrm{Aut}(X)$ we denote by $H(\theta)$ the subgroup of all finite permutations $g \in \mathfrak{S}_\infty$ which commute with θ. We define unitary representations $U^{\theta,\chi}$ as the induced representations $\mathrm{Ind}_{H(\theta)}^{\mathfrak{S}_\infty}\chi$, where χ is a unitary character of $H(\theta)$. The results in [3] are restricted to particular automorphisms θ to discuss their irreducibility and equivalence. In the present paper, we first determine the class of automorphisms $\theta \in \mathrm{Aut}(X)$ for which the unitary representation $U^{\theta,\chi}$ is irreducible. Next we give a complete classification of the irreducible representations $U^{\theta,\chi}$.

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