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STABLE BASE CHANGE FOR SPHERICAL FUNCTIONS

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To the memory of Takuro Shintani

§ 0. Introduction

Let E/F be an unramified cyclic extension of local non-archimedean fields, G a connected reductive group over F, $K(F)$ (resp. $K(E)$) a hyperspecial maximal compact subgroup of $G(F)$ (resp. $G(E)$), and $H(F)$ (resp. $H(E)$) the Hecke convolution algebra of compactly-supported complex-valued $K(F)$ (resp. $K(E)$)-biinvariant functions on $G(F)$ (resp. $G(E)$). Then the theory of the Satake transform defines (see § 2) a natural homomorphism $H(E) \to H(F)$, $\phi \to f$. There is a norm map N from the set of stable twisted conjugacy classes in $G(E)$ to the set of stable conjugacy classes in $G(F)$; it is an injection (see [Ko]). Let $\Phi'(x, f)$ denote the stable orbital integral of f in $H(F)$ at the class x, and $\Phi'(y, \phi)$ the stable twisted orbital integral of ϕ in $H(E)$ at the class y. Note that the set of regular elements in $G(F)$ is dense, and the orbital integral $\Phi(x, f)$ is uniquely determined by its restriction to the regular set. We prove

THEOREM. *If ϕ maps to f, then $\Phi'(x, \phi) = \Phi'(Nx, f)$ for any twisted stable conjugacy class x in $G(E)$ with a norm Nx regular in $G(F)$, and $\Phi'(y, f) = 0$ for any regular class y in $G(F)$ which is not of the form Nx.*

This Theorem, which is sometimes called the Fundamental Lemma, is important for the study of the Saito-Shintani base-change lifting of automorphic forms using the trace formula. In the special case where G is the unitary group $U(3)$ in three variables the Theorem becomes Lemma 3.3 of [UP], which we used in [UA]. The Theorem has obvious applications also to the study of both base-change and endoscopic liftings in the context of general unitary groups, as suggested by the work of [UP], [UA] in the case of $U(3)$, as well as by [U(2)]. In particular, it is used in

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