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EQUIVALENT CONDITIONS FOR THE TIGHTNESS OF A SEQUENCE OF CONTINUOUS HILBERT VALUED MARTINGALES

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1. Introduction

In [1] D. Aldous gave a sufficient condition for the tightness of a sequence $(X^n)_{n\geq 0}$ of right continuous (with left limits) processes taking their values in a separable complete metric space S. As already noted by Aldous this condition is far from being necessary when the *processes* X^n *are not* continuous. More precisely the Aldous-condition implies the left-quasi-continuity of all the weak limits of the sequence $(X^n)_{n\geq 0}$. (see [1] or [4]).

When the X^n's are real square integrable martingales (or more generally locally square integrable martingales), it has been shown by R. Rebolledo ([9, see also an exposition in [4]) that the Aldous-condition for the positive increasing Meyer-processes $(\langle X^n\rangle)$ implies the Aldous-condition for $(X^n)_{n\geq 0}$.

In the case of Hilbert valued martingales it has been shown in [6] that the Aldous-condition on $(\langle X^n\rangle)$ plus a tightness condition on the sequence $(\langle\langle X^n\rangle\rangle_T)_{n\geq 0}$ of operator valued random variables, $\langle\langle X^n\rangle\rangle$ being the "tensor-Meyer-process" of X^n (see [7]), is also sufficient for the tightness of $(X^n)_{n\geq 0}$.

But in general neither the Aldous-condition on $(\langle\langle X^n\rangle\rangle)_{n\geq 0}$ is necessary for the tightness of $(X^n)_{n\geq 0}$, nor the tightness of $(\langle\langle X^n\rangle\rangle)_{n\geq 0}$ alone implies the tightness of $(X^n)_{n\geq 0}$ (see J. Jacod, J. Mémin, M. Métivier [3]) *unless* some condition is assumed on the limits of the laws of the processes $\langle\langle X^n\rangle\rangle$. When the processes are real or finite dimensional, the fact that the limiting laws are carried by the subset of continuous paths in $D(\boldsymbol{R}_+, H)$ is sufficient. (see R. Rebolledo [9] and also [3] Theorem 1).

Considering only continuous processes, S. Nakao ([8]) recently proved

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