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DISTRIBUTIVE AND RELATED IDEALS IN GENERIC EXTENSIONS

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§ 0. Introduction

Let κ be a regular uncountable cardinal and I a κ-complete ideal on κ. In [11] Kanai proved that the μ-distributivity of the quotient algebra $P(\kappa)/I$ is preserved under κ-c.c. μ-closed forcing. In this paper we extend Kanai's result and also prove similar preservation results for other naturally occurring forms of distributivity. We also consider the preservation of two game theoretic properties of I and in particular, using a game theoretic equivalent of precipitousness we give a new proof of Kakuda's theorem ([10]) that the precipitousness of I is preserved under κ-c.c. forcing.

Our set theoretical notation and terminology is reasonably standard. Throughout κ will denote a regular uncountable cardinal, I a proper non-principal κ-complete ideal on κ (in the sense of [2, p. 7]), and P a forcing notion. We will only be interested in the case when $\Vdash_P$ "κ is regular and I generates an ideal on κ", and hence throughout we assume that $\Vdash_P$ "$\forall x$ (if $x \subseteq V$ and $|x| < \kappa$ then there exists a $y \in V$ such that $|y|^V < \kappa$ and $x \subseteq y$)". J will denote the P name for the ideal on κ generated by I.

DEFINITION. P is (μ, η, λ)-distributive iff whenever $p \in P$ and $\langle W_\alpha | \alpha < \mu \rangle$ is a sequence of maximal antichains below p in P, each of cardinality $\leqslant \lambda$, there is a $q \in P$ such that $q \leqslant p$ and for each $\alpha < \mu$, $|\{r \in W_\alpha | r$ is compatible with $q\}| \leqslant \eta$. P is said to be (μ, η, ∞)-distributive if it is (μ, η, λ)-distributive for each λ. We omit the η in the case $\eta = 1$.

I is said to be (μ, η, λ)-distributive iff the Boolean algebra $P(\kappa)/I$ is (μ, η, λ)-distributive.

§ 1. (μ, κ, ∞) and (μ, ∞)-distributivity

In this section we make use of generic ultrafilters to prove some preservation results for (μ, κ, ∞) and (μ, ∞)-distributivity.

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