T. Higa
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ON THE EQUIVALENCE PROBLEM AND INTEGRATION OF DIFFERENTIAL SYSTEMS

TATSUO HIGA

Introduction

The purpose of the presnet paper is to study the relationship between the theory of Lie pseudogroups and the problem of integration of differential systems (cf. [6] pp. 30–47).

Let $\mathfrak{G}$ be a Lie pseudogroup on a manifold M and S a differential system on M. Let $\mathfrak{G}(S)$ denote the largest subpseudogroup of $\mathfrak{G}$ leaving S invariant. Then the problems to be considered may be stated as follows.

A) Classify differential systems on M under the action of $\mathfrak{G}$.

B) For each differential system S on M, determine the structure of $\mathfrak{G}(S)$.

C) Using the structure of $\mathfrak{G}(S)$, reduce the problem of integration of S to that of some auxiliary differential systems, each of which is invariant under the action of a Lie pseudogroup and irreducible in a sense.

To study these problems, we use the theory of Lie pseudogroups which is developed in [7]. The problems A) and B) are subordinate to the so-called general equivalence problem (see [2] §§ 11–13). The problem C) is motivated by the classical scheme of S. Lie for the problem of integration (see [8] and [9] Introduction).

In Section 1, we recall briefly the theory of Lie pseudogroups. A Cartan system is a pair (P, C) consisting of a manifold P and an "invariant system" C on P. We can define an effective action of (P, C) on a manifold M. Then the action yields a Lie pseudogroup $\mathfrak{G}$ on M. (P, C) is called a defining Cartan system of $\mathfrak{G}$.

In Section 2, we shall study the equivalence problem of Pfaffian (differential) systems. Let (P, C) and $\mathfrak{G}$ be as above. For each Pfaffian system S on M, we construct a Cartan system $(P, C(S))$ in such a way that $(P, C(S))$ is a defining Cartan system of $\mathfrak{G}(S)$ (Theorem 2.3). Then, using $(P, C(S))$, we can study the structure of $\mathfrak{G}(S)$. Moreover, we prove the