TANGENT VECTORS TO SETS IN THE THEORY OF GEODESICS

DUMITRU MOTREANU

Introduction

In the setting of Banach manifolds the notion of tangent vector to an arbitrary closed subset has been introduced in [11] by the author and N. H. Pavel, and it has been used in flow-invariance and optimization ([11], [12], [13]). For detailed informations on tangent vectors to closed sets (including historical comments) we refer to the recent book of N. H. Pavel [17].

The aim of this paper is to apply this general concept of tangency in the study of geodesics. We are concerned with geodesics which have either the endpoints in given closed subsets or the same angle for a fixed closed subset. This approach allows one to extend important results due to K. Grove [4] and T. Kurogi ([6], [7]).

Section 1 is devoted to the general theory of tangent vectors. Here we prove some useful results in critical point theory and existence for minimization.

In Section 2 we deal with geodesics as critical points of the energy functional relative to certain sets of paths. In this way we obtain characterizations for various geodesics between closed subsets. In addition, we point out that the energy functional satisfies a generalized version of Condition (C) relative to some sets (not necessarily submanifolds).

Section 3 contains the extension of the Kurogi's work on geodesics with the same angle. More precisely, we provide sufficient conditions insuring the existence of geodesics invariant with respect to an isometry and acrossing with the same angle a compact subset.