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SOME CHARACTERIZATION AND DISTORTION THEOREMS INVOLVING FRACTIONAL CALCULUS, GENERALIZED HYPERGEOMETRIC FUNCTIONS, HADAMARD PRODUCTS, LINEAR OPERATORS, AND CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS*

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By using a certain linear operator defined by a Hadamard product or convolution, several interesting subclasses of analytic functions in the unit disk are introduced and studied systematically. The various results presented here include, for example, a number of coefficient estimates and distortion theorems for functions belonging to these subclasses, some interesting relationships between these subclasses, and a wide variety of characterization theorems involving a certain functional, some general functions of hypergeometric type, and operators of fractional calculus. Some of the coefficient estimates obtained here are fruitfully applied in the investigation of certain subclasses of analytic functions with fixed finitely many coefficients.

§1. Introduction and Definitions

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = \sum_{n=0}^{\infty} a_{n+1} z^{n+1} \qquad (a_1 = 1) \tag{1.1}$$

which are *analytic* in the unit disk $\mathcal{U} = \{z: |z| < 1\}$. Then a function $f(z)$ belonging to $\mathcal{A}$ is said to be in the class $\mathcal{V}(\alpha)$ if it satisfies the inequality

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > -\alpha \qquad (z \in \mathcal{U}) \tag{1.2}$$

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