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Nagoya Math. J.
Vol. 105 (1987), 169-186

# ON SOME DIMENSION FORMULA FOR AUTOMORPHIC FORMS OF WEIGHT ONE, II

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## § 0. Introduction

Let $\Gamma$ be a fuchsian group of the first kind and assume that $\Gamma$ contains the element $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $(=-I)$, and let $\chi$ be a unitary representation of $\Gamma$ of degree 1 such that $\chi(-I)=-1$. Let $S_1(\Gamma, \chi)$ be the linear space of cusp forms of weight one on the group $\Gamma$ with character $\chi$. We shall denote by $d_1$ the dimension of the linear space $S_1(\Gamma, \chi)$. It is not effective to compute the number $d_1$ by means of the Riemann-Roch theorem. Because of this reason, it is an interesting problem in its own right to determine the number $d_1$ by some other method (for example, [5]).

When the group $\Gamma$ has a compact fundamental domain in the upper half plane $S$[1], we have obtained the following dimension formula which is a slightly modified form of the previous result ([1]):

$$(1) \qquad d_1 = \frac{1}{2} \sum_{\{M\}} \frac{\chi(M)}{[\Gamma(M): \pm I]} \frac{\bar{\zeta}}{1-\bar{\zeta}^2} + \frac{1}{2} \operatorname*{Res}_{s=0} \zeta^*(s)\,,$$ [2]

where the sum over $\{M\}$ is taken over the distinct elliptic conjugacy classes of $\Gamma/\{\pm I\}$, $\Gamma(M)$ denotes the centralizer of $M$ in $\Gamma$, $\zeta$ is one of the eigenvalues of $M$, and $\zeta^*(s)$ denotes *the Selberg type zeta-function* defined by

$$\zeta^*(s) = \sum_{\alpha=1}^{\infty} \sum_{k=1}^{\infty} \frac{\chi(P_\alpha)^k \log \lambda_{0,\alpha}}{\lambda_{0,\alpha}^k - \lambda_{0,\alpha}^{-k}} |\lambda_{0,\alpha}^k + \lambda_{0,\alpha}^{-k}|^{-s}\,.$$

Here, $\lambda_{0,\alpha}$ denotes the eigenvalue $(\lambda_{0,\alpha}>1)$ of a representative $P_\alpha$ of the primitive hyperbolic conjugacy classes $\{P_\alpha\}$ in $\Gamma/\{\pm I\}$.

Received November 9, 1985.
Revised September 12, 1986.

1) In this case, $S_1(\Gamma, \chi)$ denotes simply the space of all holomorphic automorphic forms of weight one with $\chi$.

2) For this modified formula of $d_1$, refer to Hiramatsu ([6], Remark 1 in § 2).