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FOR WHICH FINITE GROUPS G IS THE LATTICE $\mathscr{L}(G)$ OF SUBGROUPS GORENSTEIN?

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Introduction

Let G be a finite group and $\mathscr{L}(G)$ the lattice consisting of all subgroups of G. It is well known that $\mathscr{L}(G)$ is distributive if and only if G is cyclic (cf. [2, p. 173]). Moreover, the classical result of Iwasawa [8] says that $\mathscr{L}(G)$ is pure if and only if G is supersolvable. Here, a finite lattice is called pure if all of maximal chains in it have same length and a finite group G is called supersolvable if $\mathscr{L}(G)$ has a maximal chain which consists of normal subgroups of G.

On the other hand, some remarkable connections between commutative algebra and combinatorics have been discovered in recent years. One of the main topics in this area is the concept of Cohen-Macaulay and Gorenstein posets. See, for examples, Hochster [7] and Stanley [11].

Now, with the help of Stanley [10] and Iwasawa [8], Björner [3] proved that $\mathscr{L}(G)$ is Cohen-Macaulay if and only if G is supersolvable. So, it is natural to ask for which finite groups G the lattice $\mathscr{L}(G)$ is Gorenstein.

The purpose of this paper is to prove the following

THEOREM. *Let G be a finite group and $\mathscr{L}(G)$ its lattice of subgroups. Then $\mathscr{L}(G)$ is Gorenstein if and only if G is a cyclic group whose order is either square-free or a prime power.*

§ 1. Preliminaries from group theory, commutative algebra and combinatorics

We here summarize basic definitions and results on group theory, commutative algebra and combinatorics.

(1.1) Let G be a finite group whose order $\sharp(G)$ is $p_1 p_2 \cdots p_m$, where

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