S. Takenaka
Nagoya Math. J.
Vol. 105 (1987), 19-31

REPRESENTATION OF EUCLIDEAN RANDOM FIELD

SHIGEO TAKENAKA

P. Lévy introduced a notion of Brownian motion $\mathscr{X} \equiv \{X(p); p \in M\}$ with parameter in a metric space (M, d), which is a centered Gaussian system satisfying

$$E|X(p) - X(q)|^2 = d(p, q) \quad \text{and} \quad X(O) = 0, \ O \text{ being the origin} .$$

In the case of $M = \boldsymbol{R}^n$, S^n or the hyperbolic space $\boldsymbol{H}^n$ with usual geodesic metric, the Brownian motion above has the following representation

(1) $X(p) = Y(S_p)$, where $S_p = \{\text{hyperplanes intersect } \overline{Op}\}$ and $\mathscr{Y} = \{Y(\cdot)\}$ is the Gaussian random measure associated with a certain measure μ on the set of all hyperplanes.

In this paper we shall discuss Brownian motion that corresponds to a general metric. When the metric d on $\boldsymbol{R}^n$ is expressible as $d(p, q) = r(|p - q|)$ where r is a positive increasing continuous function, the Brownian motion is called a Euclidean random field (ERF). The main purpose of this paper is to establish the representations of the form (1) for some important classes of ERFs.

In Section 1 we will consider a simple and basic class of ERFs, denote it by $\{\mathscr{U}^\rho\}$, and their representations. The covariance function of the field $\mathscr{U}^\rho$ is of finite range and rotationally invariant. The form (1) for the ordinary Brownian motion with parameter $\boldsymbol{R}^n$ is obtained as the limit $\rho \to \infty$ of these fields $\mathscr{U}^\rho$.

In Section 2 the representation of type (1) will be considered for general ERF related to the $\{\mathscr{U}^\rho\}$. We will start with a special class of ERFs. If $r(t) = t^\alpha$ the random field is called the self-similar Euclidean random field (SERF) of index α. The representation of SERF of index $0 < \alpha < 1$ will be written as a superposition of the fields $\mathscr{U}^\rho$. For general ERF, two sufficient conditions for the existence of the representation of form (1) will be given as conditions on the function $r(\cdot)$.

Received June 19, 1985.