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GENERAL NÉRON DESINGULARIZATION AND APPROXIMATION

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§1. Introduction

Let A be a noetherian ring (all the rings are supposed here to be commutative with identity), $\mathfrak{a} \subset A$ a proper ideal and $\hat{A}$ the completion of A in the $\mathfrak{a}$-adic topology. We consider the following conditions

(WAP) Every finite system of polynomial equations over A has a solution in A iff it has one in $\hat{A}$.

(AP) Every finite system of polynomial equations over A has its set of solutions in A dense with respect to the $\mathfrak{a}$-adic topology in the set of its solutions in $\hat{A}$, i.e. for every solution $\hat{y}$ from $\hat{A}$ and every positive integer $c \in N$ there exists a solution y_c in A such that $y_c \equiv \hat{y} \bmod \mathfrak{a}^c \hat{A}$.

(SAP) For every finite system of polynomial equations f over A there exists a function ν: $N \to N$ with the following property:

"For every positive integer c and for every system of elements $\tilde{y}$ from A which form a solution of f modulo $\mathfrak{a}^{\nu(c)}$ there exists a solution y_c of f in A such that $\tilde{y} \equiv y_c \bmod \mathfrak{a}^c$".

It is easy to note that (SAP) is stronger than (AP) and (WAP) is equivalent with (AP) (see (3.3.1)). We call $(A, \mathfrak{a})$ *an (AP)-couple* (resp. *(SAP)-couple*) if it satisfies (AP) (resp. (SAP)). A local ring $(A, \mathfrak{m})$ is called a ring with the property of Artin approximation or an AP-ring if the couple $(A, \mathfrak{m})$ is an AP-couple.

Examples of (non complete) AP-rings are mainly given by the following Theorems:

(1.1) THEOREM (M. Artin [A_1]). *The convergent power series rings in $X = (X_1, \cdots, X_n)$ over a non-trivial valued field of characteristic zero are* AP-*rings.*

(1.2) THEOREM (M. Artin [A_2]). *The henselization of a local ring which*

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