EXISTENCE OF NORMAL MEROMORPHIC FUNCTIONS WITH A PERFECT SET AS THE SET OF ESSENTIAL SINGULARITIES

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§1. Introduction

1. We are interested in whether there is a Cantor set E admitting no exceptionally ramified or normal meromorphic functions with E as the set of essential singularities. As for an exceptionally ramified meromorphic function, we [2] have recently given the following result.

THEOREM A. *Let E be a Cantor set with successive ratios ξ_n satisfying the condition*

$$\xi_{n+1} = o(\xi_n^5),$$

then the domain complementary to E admits no exceptionally ramified meromorphic functions with E as the set of essential singularities.

However, for a normal meromorphic function, S. Toppila [4] proved that if the set F is an infinite closed set, there exists a normal meromorphic function in the domain F^c complementary to F with at least one essential singularity in F. In [4], he gave a normal meromorphic function in F^c with one essential singularity in F.

In this paper, using the analogous method in S. Toppila [4], we shall give a normal meromorphic function with a Cantor set as the set of essential singularities.

Our result is stated as follows:

THEOREM. *Let E be a Cantor set with successive ratios ξ_n such that*

$$(1) \qquad \lim_{n \to \infty} \xi_n = 0$$

and

$$(2) \qquad \xi_{n+1} = O(\xi_n).$$