N. V. Trung
Nagoya Math. J.
Vol. 102 (1986), 1-49

TOWARD A THEORY OF GENERALIZED COHEN-MACAULAY MODULES

NGÔ VIÊT TRUNG

Introduction

Throughout this paper, A denotes a noetherian local ring with maximal ideal $\mathfrak{m}$ and M a finitely generated A-module with $d := \dim M \geqq 1$.

DEFINITION. M is called a generalized Cohen-Macaulay (abbr. C-M) module if

$$l(H^i_{\mathfrak{m}}(M)) < \infty$$

for $i = 0, \cdots, d-1$, where l denotes the length and $H^i_{\mathfrak{m}}(M)$ the ith local cohomology module of M with respect to $\mathfrak{m}$.

The notion of generalized C-M modules was introduced in [6]. It has its roots in a problem of D.A. Buchsbaum. Roughly speaking, this problem says that the difference

$$I(\mathfrak{q}; M) := l(M/\mathfrak{q}M) - e(\mathfrak{q}; M)$$

takes a constant value for all parameter ideals $\mathfrak{q}$ of M, where $e(\mathfrak{q}; M)$ denotes the multiplicity of M relative to $\mathfrak{q}$ [5]. In general, that is not true [30]. However, J. Stückrad and W. Vogel found that modules satisfying this problem enjoy many interesting properties which are similar to the ones of C-M modules and gave them the name Buchsbaum modules [22], [23]. That led in [6] to the study of modules M with the property

$$I(M) := \sup I(\mathfrak{q}; M) < \infty$$

where $\mathfrak{q}$ runs through all parameter ideals of M, and it turned out that they are just generalized C-M modules.

The class of generalized C-M module is rather large. For instance, most of the considered geometric local rings such as the ones of isolated singularities or of the vertices of affine cones over projective curves are

Received February 15, 1983.