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THE THETA FUNCTIONS OF SUBLATTICES OF THE LEECH LATTICE

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To the memory of late Takehiko Miyata

Introduction

Let Λ be the Leech lattice which is an even unimodular lattice with no vectors of squared length 2 in 24-dimensional Euclidean space $\boldsymbol{R}^{24}$. Then the Mathieu Group M_{24} is a subgroup of the automorphism group $\cdot 0$ of Λ and the action on Λ of M_{24} induces a natural permutation representation of M_{24} on an orthogonal basis $\{e_i \mid 1 \leqq i \leqq 24\}$ of $\boldsymbol{R}^{24}$. For $m \in M_{24}$, let Λ_m be the sublattice of vectors invariant under m:

$$\Lambda_m = \{x \in \Lambda \mid x^m = x\}$$

and $\Theta_m(z)$ be the theta function of Λ_m:

$$\Theta_m(z) = \sum_{x \in \Lambda_m} e^{\pi i z \ell(x)}$$

where $\ell(x) = \ell(x, x)$ and $\ell(x, y)$ $(x, y \in \boldsymbol{R}^{24})$ is the inner product of $\boldsymbol{R}^{24}$ with $\ell(e_i, e_j) = 2\delta_{ij}$.

One of the purposes of this note is to express $\Theta_m(z)$ explicitly by the classical Jacobi theta functions $\theta_i(z)$ $(i = 2, 3, 4)$ and the Dedekind eta-function. The results are given in Table 2 of Section 2. Furthermore, by using these expressions of $\Theta_m(z)$, we will prove the following theorem:

THEOREM 2.1. *Let $\Theta_m(z)$ $(m \in M_{24})$ be as above and let*

$$\eta_m(z) = \prod_t \eta(tz)^{r_t}$$

where $\eta(z)$ is the Dedekind eta-function

$$\eta(z) = q^{1/12} \prod_{n=1}^{\infty} (1 - q^{2n}) \qquad (q = e^{\pi i z})$$

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