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HOMOGENEOUS VECTOR BUNDLES AND STABILITY

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§1. Introduction

In [5, 6, 7] I introduced the concept of Einstein-Hermitian vector bundle. Let E be a holomorphic vector bundle of rank r over a complex manifold M. An Hermitian structure h in E can be expressed, in terms of a local holomorphic frame field $s_1, \cdots, s_r$ of E, by a positive-definite Hermitian matrix function $(h_{i\bar{j}})$ defined by

$$h_{i\bar{j}} = h(s_i, s_j).$$

Then the Hermitian connection form and its curvature form are given by

$$\omega^i_j = \sum h^{i\bar{k}} d' h_{j\bar{k}},$$
$$\Omega^i_j = d'' \omega^i_j.$$

In terms of a local coordinate system $z^1, \cdots, z^n$ of M, we can write

$$\Omega^i_j = \sum R^i_{j\alpha\bar{\beta}} dz^\alpha \wedge d\bar{z}^\beta.$$

Given an Hermitian metric

$$g = 2\sum g_{\alpha\bar{\beta}} dz^\alpha d\bar{z}^\beta$$

on M, we define the g-trace K of the curvature of (E, h) by setting

$$K^i_j = \sum g^{\alpha\bar{\beta}} R^i_{j\alpha\bar{\beta}}.$$

Then K is a field of endomorphisms of E with components K^i_j. We say that (E, h, M, g) is an *Einstein-Hermitian vector bundle* if

$$K = \varphi I_E, \quad \text{i.e.,} \quad K^i_j = \varphi \delta^i_j,$$

where φ is a (real) function on M and I_E is the identity endomorphism of E.

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