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ON THREEFOLDS WITH LOW SECTIONAL GENUS

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Introduction

The general problem of rebuilding the threefolds X endowed with a given ample divisor H, possibly non-effective, is closely related to the study of the complete linear system $|K_X + H|$ adjoint to H. Many powerful results are known about $|K_X + H|$, for instance when the linear system $|H|$ contains a smooth surface or, more particularly, when H is very ample (e.g. see Sommese [S1] and [S2]). From this point of view we study some properties of $|K_X + H|$, which turn out to be very useful in the description of the threefolds X polarized by an ample divisor H whose arithmetic virtual genus $g(H)$ is sufficiently low.

In section 1 we recall some preliminaries.

In section 2 an explicit description of the threefolds on which the sheaf $\mathcal{O}_X(n(K_X + H))$ fails to be spanned by its global sections for $n \gg 0$ is pointed out (Theorem 2.2). This is a consequence of a part of Mori's theory on extremal rays. In brief this theorem assures that it is always possible to assume $\mathcal{O}_X(n(K_X + H))$ to be spanned for $n \gg 0$ up to contractions of (-1)-planes, apart from some classes of threefolds, which are fully described. Incidentally this answers a question put by Sommese in [S2].

In section 3 first of all we characterize the pairs X, H in the cases $g(H) = 0$ and $g(H) = 1$ (Propositions 3.1 and 3.2), in this way recovering some classical results (cf. [E]; see also [Ro], IV, § 9) in the wider context of ample divisors. As far as higher values of $g(H)$ are concerned, the pairs X, H are classified under the assumption $H^3 \geq 2g(H) - 2$ (Theorem 3.3). This analysis together with the general theory of Fano threefolds leads to the explicit description of X, H where $g(H) = 2$, $H^3 \geq 2$ (Proposition 3.5). This is again linked to a classical piece of research carried out by Enriques in [E].

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