S. E. Graversen and M. Rao
Nagoya Math. J.
Vol. 100 (1985), 163-180

# QUADRATIC VARIATION AND ENERGY

S. E. GRAVERSEN AND M. RAO

## § 0. Introduction

It is well known that the concept of energy has played a fruitful role in potential theory and Markov processes. Cartan's work [6] led to kernel-free potential theories of Beurling-Deny [2]. Since then many authors have worked on this, M. Fukushima [8], M. Silverstein [16], J. Bliedner [3], Berg-Forst [1], to name some. In these works, however, the main thrust is Hilbert space theoretic.

In the martingale context the notion of energy was introduced by Meyer [11]. This has been applied in the potential theory of general Hunt processes in Z. R. Pop-Stojanovic and M. Rao [14], S. E. Graversen and M. Rao [10].

In a different direction G. Brosamler [5] in one original paper initiated the study of quadratic variation in the context of potential theory. Brosamler's formula suggests an intimate connection with energy and is indeed the inspiration for the present work.

Let us briefly describe the results. In Section 1 we define what we mean by the energy of a cadlag process, i.e. a right continuous process with left limits. Then we show that such a process is a sum of a martingale and a previsible process with a special "orthogonality to martingales" property. Uniqueness of this decomposition, its relation to standard decompositions and an Itô formula for $C^1$-functions are then discussed. In this connection we mention the work of Föllmer [9].

Section 2 deals with energy of potentials in the context of Hunt processes. There are at least two different definitions to be found in the literature [8, 14]. We discuss the relation of these with the energy of the process obtained by composing the process with the potential.

Section 3 deals with application of the preceding to classical potentials in bounded domains of the Euclidean space. We obtain the well-

Received September 3, 1984.