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GENERAL NÉRON DESINGULARIZATION

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§1. Introduction

Let $R \subset R'$ be an "unramified" extension of discrete valuation rings in the sense that a local parameter p of R is also a local parameter in R'. Suppose that the inclusion $R \to R'$ induces separable extensions on fraction and residue fields. Then

(1.1) THEOREM (Néron [N]). *R' is a filtered inductive limit of its finite type smooth sub-R-algebras.*

In other words every finite type sub-R-algebra B of R' can be embedded in a smooth finite type sub-R-algebra B' of R'.

Let T be a valuation ring containing a field k of characteristic zero. Then

(1.2) THEOREM (Zariski [Z]). *T is a filtered inductive limit of the finite type smooth sub-k-algebras.*

Actually the above result is stated only in the case when the fraction field of T is an algebraic function field over k; but always we can reduce the problem to this case because the fraction field of T is certainly a filtered inductive union of algebraic function fields over k.

Theorems as above are very useful when we want to reduce the solvability in R' (resp. T) of some polynomial equations over R (resp. k) to the solvability of some polynomial equations for which it is possible to apply the Implicit Function Theorem. For this reason many results of Artin approximation theory are based on them. Attempts for extensions of these theorems were made in [KMPPR] Ch. V, [P_1], [CP] and [P_2] but they preserved too much from Néron's case. In [P_2] (4.2.1) we put the following:

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