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CONSTRUCTION OF SIEGEL MODULAR FORMS OF DEGREE THREE AND COMMUTATION RELATIONS OF HECKE OPERATORS

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In connection with the Shimura correspondence, Shintani [6] and Niwa [4] constructed a modular form by the integral with the theta kernel arising from the Weil representation. They treated the group $Sp(1) \times O(2,1)$. Using the special isomorphism of $O(2,1)$ onto $SL(2)$, Shintani constructed a modular form of half-integral weight from that of integral weight. We can write symbolically his case as "$O(2,1) \to Sp(1)$". Then Niwa's case is "$Sp(1) \to O(2,1)$", that is from the half-integral to the integral. Their methods are generalized by many authors. In particular, Niwa's are fully extended by Rallis-Schiffmann to "$Sp(1) \to O(p,q)$".

In [7], Yoshida considered the Weil representation of $Sp(2) \times O(4)$ and constructed a lifting from an automorphic form on a certain subgroup of $O(4)$ to a Siegel modular form of degree two. In this note, under the spirit of Yoshida, we consider $Sp(3) \times O(4)$ and construct a Siegel modular form of degree three. We use Kashiwara-Vergne's results [2] for the analysis of the infinite place. Roughly speaking, the representation (λ, V_λ) of $O(4)$ which corresponds to an irreducible component of the Weil representation determines the representation $\tau(\lambda)$ of $GL(3, C)$. Then we can make the $V_{\lambda'}$-valued theta series. By integrating the inner product of this theta series and a V_λ-valued automorphic form, we get a Siegel modular form (Proposition 1). The main results in this note are commutation relations of Hecke operators (Theorems 1, 2). By these formulas we can express the Andrianov's L-function by the product of the L-functions of original forms. It is desired that the relations of Theorems 1 and 2 are computed more naturally.

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