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GROMOV'S CONVERGENCE THEOREM AND ITS APPLICATION

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One of the basic questions of Riemannian geometry is that "If two Riemannian manifolds are similar with respect to the Riemannian invariants, for example, the curvature, the volume, the first eigenvalue of the Laplacian, then are they topologically similar?". Initiated by H. Rauch, many works are developed to the above question. Recently M. Gromov showed a remarkable theorem ([7] 8.25, 8.28), which may be useful not only for the above question but also beyond the above. But it seems to the author that his proof is heuristic and it contains some gaps (for these, see § 1), so we give a detailed proof of 8.25 in [7]. This is the first purpose of this paper. Second purpose is to prove a differentiable sphere theorem for manifolds of positive Ricci curvature, using the above theorem as a main tool.

For a d-dimensional Riemannian manifold M, we denote by K_M the sectional curvature, by $\mathrm{vol}(M)$ the volume, by $\mathrm{diam}(M)$ the diameter, by $d_M(m, n)$ the distance between m and n induced from Riemannian metric g and by i_M the injectivity radius.

A subset B is called δ-dense when for any point $m \in M$, there exists a point $n \in B$ with $d_M(m, n) \leqq \delta$. A subset B is called δ-discrete if $n_1, n_2 \in B$ $(n_1 \neq n_2)$ implies $d_M(n_1, n_2) \geqq \delta$. Let $M(d, \Lambda, i_0)$ (resp. $M(d, \Lambda, \rho, v)$) be the category of all complete Riemannian manifolds M with dimension $= d$, $|K_M| \leqq \Lambda$ and $i_M \geqq i_0$ (resp. dimension $= d$, $|K_M| \leqq \Lambda$, $\mathrm{diam}(M) \leqq \rho$, $\mathrm{vol}(M) \geqq v$).

The following theorem is seemingly different from 8.25 in [7] but the inwardness is essentially same.

THEOREM 1 (Gromov's convergence theorem). *Given* $d, \Lambda, i_0 > 0$, $0 < R < \min(1/2\sqrt{\Lambda}, i_0/2)$, *for any* $\delta > 0$, *there exist* $a = a(d, \Lambda, i_0, R; \delta) > 0$ *and*

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