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CLASSIFICATION OF ALGEBRAIC NON-RULED SURFACES WITH SECTIONAL GENUS LESS THAN OR EQUAL TO SIX

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Introduction

In this paper we have given a biholomorphic classification of smooth, connected, projective, non-ruled surfaces X with a smooth, connected, hyperplane section C relative to L, where L is a very ample line bundle on X, such that $g = g(C) = g(L)$ is less than or equal to six. For a similar classification of rational surfaces with the same conditions see [Li]. From the adjunction formula

$$2g - 2 = L \cdot (K_X + L)$$

where K_X denotes the canonical line bundle on X. In the cases in which $g = 0, 1$, X is rational or ruled, see [Na]. Thus we have to examine only the cases in which $g = 2, \cdots, 6$. Our classification goes far beyond the birational classification of Leonard Roth [Ro] and the one that Polkin Ionescu [Io] has given for $g \leq 4$, Our main tool is the adjunction process and the results of Andrew John Sommese which are in [So]. Our notations are as in [So] except for the following. X will denote a smooth, connected, projective, non-ruled surface and L a very ample line bundle on X. Let $\bar{L} = K_X \otimes L$. Then $h^{1,0}(X) \neq g$ [So, (1.5.2)], and $\bar{L}$ is generated by global sections [So, (1.5)], and the induced morphism ϕ has a 2-dimensional image [So, (2.0.1) and (2.1)]. Let $\hat{X} \to X$ be the Stein factorization and π: $\hat{X} \to X$ the induced morphism. Then $\hat{X}$ is a non singular projective surface, π is the blow-up at a finite number of points of $\hat{X}$, $\hat{L} = \pi_* L$ and $K_{\hat{X}} + \hat{L}$ are ample, and $K_X + L \sim \pi^*(K_{\hat{X}} + \hat{L})$ [So, (2.3)]. We call $(\hat{X}, \hat{L})$ the minimal pair of (X, L). Our main goal is to classify the pairs $(\hat{X}, \hat{L})$ and eventually the pairs (X, L). Let $d = (L^2)$, $g = g(L)$, $\hat{d} = (\hat{L}^2)$, $d' = (L'^2)$,

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