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Nagoya Math. J.
Vol. 135 (1994), 113–120

THE MOMENTS OF THE ZETA-FUNCTION ON THE LINE $\sigma = 1$

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1. Introduction

The evaluation of the integral

$$\int_1^T |\zeta(\sigma + it)|^{2k}\, dt \quad (\sigma \in \mathbf{R},\ k \in \mathbf{R}^+ \text{ fixed}) \tag{1.1}$$

represents one of the fundamental problems of the theory of the Riemann zeta-function (see [4] for a comprehensive account). In view of the functional equation

$$\zeta(s) = \chi(s)\zeta(1-s),\ \chi(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s) \asymp |t|^{\frac{1}{2}-\sigma} \quad (s = \sigma + it)$$

it is clear that one has to distinguish between the following three principal cases:

a) $\sigma = 1/2$ ("the critical line"),
b) $1/2 < \sigma < 1$ ("the critical strip"),
c) $\sigma = 1$.

Although the cases a) and b), which have countless applications to various branches of number theory, have been extensively studied in the literature, it seems that the case c) has been somewhat neglected. Thus it is only recently that R. Balasubramanian et al. [1] have obtained precise results for the case c) if $k = 1$, which is the most important case. If, for $T > 3$, one defines the function $R(T)$ by the formula

$$\int_1^T |\zeta(1 + it)|^2\, dt = \zeta(2)T - \pi \log T + R(T), \tag{1.2}$$

then it was proved in [1] that

$$R(T) = O(\log^{2/3} T(\log\log T)^{1/3}), \tag{1.3}$$

Received April 15, 1992.
*) Research financed by the Mathematical Institute of Belgrade