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DECOMPOSITION PROBLEM OF PROBABILITY MEASURES RELATED TO MONOTONE REGULARLY VARYING FUNCTIONS

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1. Introduction

This paper deals with a decomposition problem for some classes of distributions. Let $\mathbf{D}$ be a given class of distribution on $\mathbf{R}^1$, which we are interested in. After showing that the class $\mathbf{D}$ is closed under convolution, our purpose is to give an answer to the inverse problem: if the convolution of two distributions μ_1 and μ_2 belongs to $\mathbf{D}$, then do μ_1 and μ_2 belong to $\mathbf{D}$?

Such an inverse problem is solved affirmatively for the class of Gaussian distributions, the class of Poisson distributions and the class of convolutions of Gaussian and Poisson ([5]). In this paper, we study this decomposition problem for several classes characterized by regular variation. A positive measurable function f is said to be regularly varying (r.v.) with index $\rho(\in \mathbf{R}^1)$ if $\lim_{x\to\infty} f(kx)/f(x) = k^\rho$ for each $k > 0$. In particular, f is called slowly varying (s.v.) if $\rho = 0$. It is well-known that the domain of attraction of Gaussian distribution (denoted by $\mathbf{D}_2$) is identical with the class of distributions whose truncated variances $\int_{|t|<x} t^2\mu(dt)$ are s.v. Concerning the inverse problem for $\mathbf{D}_2$, the author shows in [7] that there exist two distributions μ_1 and μ_2 such that neither μ_1 nor μ_2 belongs to $\mathbf{D}_2$ but the convolution of μ_1 and μ_2 belongs to $\mathbf{D}_2$. The proof depends on the fact that there is a non-decreasing s.v. function that is represented as the sum of positive non-decreasing functions that are not s.v.

We investigate the class $\mathbf{D}(\alpha)$ of distributions on $[0, \infty)$ with r.v. tails with index $-\alpha$ for $\alpha \geq 0$ and the class $\mathbf{C}$ of distributions on $[0, \infty)$ with s.v. truncated means. These classes are related to various limit theorems: the domain of attraction of stable laws, relative stability, the ratio of maximum to sum of an i.i.d.

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