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ON VANISHING OF THE TWISTED RATIONAL DE RHAM COHOMOLOGY ASSOCIATED WITH HYPERGEOMETRIC FUNCTIONS

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Introduction

Recent development in hypergeometric functions in several variables has made the importance of studying twisted rational de Rham cohomology clear to many specialists. Roughly speaking, a hypergeometric function in our sense is the integral of a product of complex powers of polynomials $P_j(u_1, \ldots, u_n) : \int U du_1 \wedge \cdots \wedge du_n$, $U = \Pi P_j^{\alpha_j}$, integration being taken over some cycle. So we are led naturally to consider the twisted rational de Rham cohomology, which is a direct generalization of the usual de Rham cohomology to multivalued case. Thus it will be useful to give the exposition of twisted de Rham cohomology, which is necessary to the study of hypergeometric functions and this paper aims at the point. In the paper, we shall consider also the logarithmic complex, which is a very important subcomplex of the twisted rational de Rham complex and is quasi-isomorphic to the complex in many good cases. We explain the content of the paper in more details; Let D be the divisor of $\mathbf{C}^n$ defined by ΠP_j and set $\omega = dU/U$. In §1 and §2 we shall show some basic properties of logarithmic forms that are necessary to our later applications. In §3 we shall treat the case where each P_j is *homogeneous* and show acyclicity of the twisted de Rham complex under the condition $\sum \alpha_j \deg P_j \notin \mathbf{Z}$. To go further to inhomogeneous case, we introduce in §4 the *degree frltration* on the logarithmic complex and compare the associated graded complex with the complex $(\Omega^{\cdot}(\bar{D}), \nabla_{\bar{\omega}})$ where $\bar{D}$ is the divisor defined by the homogeneous part $\bar{P}_j$ of P_j and $\bar{\omega} = \sum \alpha_j d\bar{P}_j / \bar{P}_j$. Using acyclicity of homogeneous case and the standard argument of filtered complex, in §5, we shall prove the vanishing theorem for twisted rational de Rham cohomology under a certain regularity conditions. In case each P_j is linear and the arrangement $\{P_j = 0\}_{1 \leq j \leq m}$ is

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