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Nagoya Math. J.
Vol. 135 (1994), 1–41

LIFTS OF SOME TENSOR FIELDS AND CONNECTIONS TO PRODUCT PRESERVING FUNCTORS

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0. Introduction

In this paper we define some lifts of tensor fields of types $(1, k)$ and $(0, k)$ as well as connections to a product preserving functor $\mathcal{F}$. We study algebraic properties of introduced lifts and we apply these lifts to prolongation of geometric structures from a manifold M to $\mathcal{F}(M)$. In particular cases of the tangent bundle of p^r-velocities and the tangent bundle of infinitesimal near points our constructions contain all constructions due to Morimoto (see [20]-[23]). In the cases of the tangent bundle our definitions coincide with the definitions of Yano and Kobayashi (see [31]). To construct our lifts and to study its properties we use only general properties of product preserving functors. All lifts verify so-called *the naturality condition.* It means that for a smooth mapping $\varphi : M \to N$ and for two φ-related geometric objects defined on M and N its lifts to $\mathcal{F}(M)$ and $\mathcal{F}(N)$ respectively are $\mathcal{F}(\varphi)$-related. We explain later the term *φ-related* for considered geometric objects.

In the presented paper we do not study problems of classifications of lifts.

A product preserving functor is a covariant functor $\mathcal{F}$ from the category of manifolds into the category of fibered manifolds such that $\mathcal{F}(M_1 \times M_2)$ is equivalent to $\mathcal{F}(M_1) \times \mathcal{F}(M_2)$. In Section 1 we formulate properties of product preserving functors used in the present paper.

Let $\mathcal{F}$ be a product preserving functor. In Section 2 we recall lifts of vector fields and functions to $\mathcal{F}$. Lifts of vector fields was introduced by Kolář in [14]. They are parametrized by elements of so-called the Weil algebra $A = \mathcal{F}(\mathbf{R})$ associated to $\mathcal{F}$. Lifts of functions to $\mathcal{F}$ was studied by Mikulski in [17]. They depend on functions $\lambda : A \to \mathbf{R}$. The defined lifts verify the naturality condition.

Let $\varphi : M \to N$ be a smooth mapping. Vector fields X, Y defined on M and N respectively are called φ-related if the following diagram

Received June 10, 1993.