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COMMUTATIVE ALGEBRAS FOR ARRANGEMENTS

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1. Introduction

Let V be a vector space of dimension l over some field $\mathbf{K}$. A hyperplane H is a vector subspace of codimension one. An arrangement $\mathscr{A}$ is a finite collection of hyperplanes in V. We use [7] as a general reference. Let $M(\mathscr{A}) = V - \cup_{H\in\mathscr{A}} H$ be the complement of the hyperplanes. Let V^* be the dual space of V. Each hyperplane $H \in \mathscr{A}$ is the kernel of a linear form $\alpha_H \in V^*$, defined up to a constant. The product

$$Q(\mathscr{A}) = \prod_{H\in\mathscr{A}} \alpha_H$$

is called a *defining polynomial* of $\mathscr{A}$. Brieskorn [3] associated to $\mathscr{A}$ the finite dimensional skew-commutative algebra $R(\mathscr{A})$ generated by 1 and the differential forms $d\alpha_H/\alpha_H$ for $H \in \mathscr{A}$. When $\mathbf{K} = \mathbf{C}$, the algebra $R(\mathscr{A})$ is isomorphic to the cohomology algebra of the open manifold $M(\mathscr{A})$. The structure of $R(\mathscr{A})$ was determined in [6] as the quotient of an exterior algebra by an ideal. In particular this shows that $R(\mathscr{A})$ depends only on the intersection poset of $\mathscr{A}$, $L(\mathscr{A})$, and not on the individual linear forms α_H.

A subarrangement $\mathscr{B} \subseteq \mathscr{A}$ is called *independent* if $\cap_{H\in\mathscr{B}} H$ has codimension $|\mathscr{B}|$, the cardinality of $\mathscr{B}$. In a special lecture at the Japan Mathematical Society in 1992, Aomoto suggested the study of the graded $\mathbf{K}$-vector space

$$AO(\mathscr{A}) = \sum_{\mathscr{B}} \mathbf{K}Q(\mathscr{B})^{-1}, \quad \mathscr{B} \text{ independent.}$$

It appears as the top cohomology group of a certain 'twisted' de Rham chain complex [1]. When $\mathbf{K} = \mathbf{R}$, he conjectured that the dimension of $AO(\mathscr{A})$ is equal to the number of connected components (chambers) of $M(\mathscr{A})$, which he proved for generic arrangements. In this paper we prove this conjecture in general. We construct a

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