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EXPLICIT FORMULAS FOR LOCAL FACTORS: ADDENDA AND ERRATA

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Introduction

In [3], the author studied certain local integrals derived from Fourier coefficient computations on Eisenstein series. Members of a family of Dirichlet series were characterized as a product of an explicit term with a mysterious polynomial factor. In a recent letter to the author, Professor Shoyu Nagaoka asked specific questions concerning the polynomial factor. Several of these questions can be answered by the techniques in [3]. In Part I of that paper, the relevant term is described precisely; however, in Part II, the term is described as a mysterious, albeit finite, sum. The present paper complete [3] by recording what little is known of that sum.

We illustrate our tables by settling one of the questions raised in Professor Nagaoka's letter. Let F is a totally real number field and let K/F be a purely imaginary quadratic extension. Let $\mathfrak{D}$ be the discriminant of K/F, and let $h \in \mathfrak{D}^{-1}$. For a finite prime $\mathcal{P}$ of F,

$$\bar{\alpha}^{(2)}_{\mathcal{P}}\left(S, \begin{bmatrix} h & 0 \\ 0 & 0 \end{bmatrix}\right) = (1 - q^{-s})(1 - \psi(\mathcal{P})q^{1-s})(1 - \psi(\mathcal{P})q^{2-s})^{-1}\left(\sum_{j=0}^{b} q^{j(3-s)}\right), \tag{1}$$

where the α-series derives from Eisenstein series for the hermitian modular group of genus 2, ψ is the ideal character of K/F (normalized to be 0 if $\mathcal{P}$ ramifies), $q = N\mathcal{P}$, and $\mathcal{P}^b$ devides the ideal $(h)\mathfrak{D}$ while $\mathcal{P}^{b+1}$ does not.

1. The α-series

Let F be a local of any characteristic except 2 and let R be (a choice of) the ring of integers of F. Let $\mathcal{P}$ be the prime of R, and put

$$q = N\mathcal{P}. \tag{2}$$

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