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THE MINIMUM AND THE PRIMITIVE REPRESENTATION OF POSITIVE DEFINITE QUADRATIC FORMS

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Let M, N be positive definite quadratic lattices over $\mathbf{Z}$ with $\mathrm{rank}(M) = m$ and $\mathrm{rank}(N) = n$ respectively. When there is an isometry from M to N, we say that M is represented by N (even in the local cases). In the following, we assume that the localization M_p is represented by N_p for every prime p. Let us consider the following assertion $\mathrm{A}_{m,n}(N)$:

$\mathrm{A}_{m,n}(N)$: *There exists a constant $c(N)$ dependent only on N so that M is represented by N if $\min(M) > c(N)$, where $\min(M)$ denotes the least positive number represented by M.*

We know that this is true if $n \geq 2m + 3$, and a natural problem is whether the condition $n \geq 2m + 3$ is the best or not. It is known that this is the best if $m = 1$. But in the case of $m \geq 2$, what we know at present, is that there is an example N so that $\mathrm{A}_{m,n}(N)$ is false if $n - m = 3$. We do not know such examples when $n - m = 4$. Anyway, analyzing the counter-example, we come to the following two assertions $\mathrm{APW}_{m,n}(N)$ and $\mathrm{R}_{m,n}(N)$.

$\mathrm{APW}_{m,n}(N)$: *There exists a constant $c'(N)$ dependent only on N so that M is represented by N if $\min(M) > c'(N)$ and M_p is primitively represented by N_p for every prime p.*

$\mathrm{R}_{m,n}(N)$: *There is a lattice M' containing M such that M'_p is primitively represented by N_p for every prime p and $\min(M')$ is still large if $\min(M)$ is large.*

If the assertion $\mathrm{R}_{m,n}(N)$ is true, then the assertion $\mathrm{A}_{m,n}(N)$ is reduced to the apparently weaker assertion $\mathrm{APW}_{m,n}(N)$. If the assertion $\mathrm{R}_{m,n}(N)$ is false, then it becomes possible to make a counter-example to the assertion $\mathrm{A}_{m,n}(N)$. As a matter of fact, the assertion $\mathrm{R}_{m,m+3}(N)$ is false in a certain kind of lattices N and it yields examples of N such that the assertion $\mathrm{A}_{m,m+3}(N)$ is false. Note that $\mathrm{APW}_{1,4}(N)$ is true for every N although $A_{1,4}(N)$ is false in general.

We proved in [4] that the assertion $\mathrm{R}_{m,2m+2}(N)$ is true if $m \geq 2$. The aim of this paper is to show that the assertion $\mathrm{R}_{m,2m+1}(N)$ is also true if $m \geq 3$ (Theorem

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