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EXISTENCE OF DIRICHLET FINITE HARMONIC MEASURES ON EUCLIDEAN BALLS

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To Professor Makoto Ohtsuka on his seventieth birthday

Divide the ideal boundary of a noncompact Riemannian manifold M into two parts δ_0 and δ_1. Viewing that M is surrounded by two conducting electrodes δ_0 and δ_1, we ask whether $(M\ ;\ \delta_0,\ \delta_1)$ functions as a condenser in the sense that the unit electrostatic potential difference between two electrodes is produced by putting a charge of finite energy on one electrode when the other is grounded. The *generalized condenser problem* asks whether there exists a subdivision $\delta_0 \cup \delta_1$ of the ideal boundary of M such that $(M\ ;\ \delta_0,\ \delta_1)$ functions as a condenser.

Mathematically the problem has two equivalent formulations one of which is in the geometric and the other in the analytic form. In the geometric formulation the problem is negatively settled if and only if the linear Royden harmonic boundary $\Delta(M)$ of M is connected. In the analytic formulation the problem is also negatively settled if and only if every Dirichlet finite harmonic measure on M reduces to a constant on M. A fairy general discussion of the generalized condenser problem for Riemannian manifolds M was carried out in [22] one of whose consequences is that the Royden harmonic boundary $\Delta(B^n)$ of the Euclidean unit ball B^n in the Euclidean space R^n of dimension $n \geq 2$ is connected which generalizes the classical result in the complex function theory that $\Delta(B^2)$ is connected (cf. e.g. [3], [1], [12], [10], [21], [25], etc.).

In this paper we discuss the above generalized condenser problem for the Euclidean ball B^n of dimension $n \geqq 2$ in a broader potential theoretic setting that the underlying harmonic structure is given by the p-Laplace equation

$$-\nabla\cdot(|\nabla u|^{p-2}\nabla u) = 0 \quad (1 < p \leqq n),$$

the solutions of which are the so-called p-harmonic functions. Here the exponent p

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