R. Salvati Manni
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ON THE PROJECTIVE VARIETIES ASSOCIATED WITH SOME SUBRINGS OF THE RING OF THETANULLWERTE

RICCARDO SALVATI MANNI

0. Introduction

Let (X, L) be a principally polarized abelian variety (ppav) of dimension g such that L is a symmetric line bundle, i.e. $i^*L \simeq L$ where i is the inversion map $i(x) = -x$. We shall denote by $X[2]$ the two torsion points of X which are fixed by i. For any x in $X[2]$ we have an isomorphism

$$t_x^*(L)^2 \simeq L^2. \tag{1}$$

Here t_x is the translation map.

Let τ be a point of the Siegel upper half space $\mathbf{H}_g$ and X be the abelian variety $\mathbf{C}^g/(\tau\mathbf{Z}^g + \mathbf{Z}^g)$. As symmetric line bundle L we take $\mathbf{C}^g \times \mathbf{C}/(\tau\mathbf{Z}^g + \mathbf{Z}^g)$ with the action $(\tau a + b)(z, w) = (z + \tau a + b, \mathbf{e}(-(1/2){}^t a\tau a + {}^t az)w)$.

Here $\mathbf{e}(t)$ stands for $\exp(2\pi it)$.

For $x = (1/2)(\tau m' + m'')$ in $X[2]$, t_x^*L is still symmetric and we have that the theta function of characteristic m and modulus τ :

$$\vartheta_m(\tau, z) = \sum_{p\in\mathbf{Z}^g} \mathbf{e}\left((1/2){}^t\left(p + \frac{m'}{2}\right)\tau\left(p + \frac{m'}{2}\right) + {}^t\left(p + \frac{m'}{2}\right)\left(z + \frac{m''}{2}\right)\right) \tag{2}$$

is up to a multiplicative constant the unique section of the above line bundle. Here m' and m'' are in $\{0, 1\}^g$.

Sometime, if it will be necessary, we shall write $\vartheta_m(\tau, z) = \vartheta\begin{bmatrix} m' \\ m'' \end{bmatrix}(\tau, z)$.

It is a well known fact that a basis of $H^0(X, L^2)$ is given by the 2^g theta function $\vartheta\begin{bmatrix} m' \\ 0 \end{bmatrix}(2\tau, 2z)$ and from (1) we have theta relation

$$\vartheta_m(\tau, z)^2 = \sum_{\sigma} \mathbf{e}((1/2){}^t(m' + \sigma)m'')\,\vartheta\begin{bmatrix} \sigma \\ 0 \end{bmatrix}(2\tau, 0)\,\vartheta\begin{bmatrix} m' + \sigma \\ 0 \end{bmatrix}(2\tau, 2z). \tag{3}$$

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