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DEPTH FORMULAS FOR CERTAIN GRADED RINGS ASSOCIATED TO AN IDEAL

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1. Introduction

In this paper, we investigate the relationship between the depths of the Rees algebra $R[It]$ and the associated graded ring $\mathrm{gr}_I(R)$ of an ideal I in a local ring (R, m) of dimension $d > 0$. Here

$$R[It] := R \oplus It \oplus I^2t^2 \oplus \cdots$$

and

$$\mathrm{gr}_I(R) := R/I \oplus I/I^2 \oplus I^2/I^3 \oplus \cdots.$$

These rings are important not only algebraically, but geometrically as well. For instance, Proj $R[It]$ is the blow-up of Spec(R) with respect to I. The relationship between the Cohen-Macaulayness of these two rings has been studied extensively. We list here a few of the many results on this relationship:

(1.1) $R[It]$ is Cohen-Macaulay for every parameter ideal I of R if and only if R is Buchsbaum and $H_m^i(R) = 0$ for $i \neq 1, d$ ([GS1]).

(1.2) Suppose R is Cohen-Macaulay with infinite residue field and let I be an m-primary ideal of R. Then $R[It]$ is Cohen-Macaulay if and only if $\mathrm{gr}_I(R)$ is Cohen-Macaulay and the reduction number of I is less than d ([GS2]).

(1.3) Let I be an ideal of R of positive height. Then $R[It]$ is Cohen-Macaulay if and only if $H_N^i(\mathrm{gr}_I(R))_n = 0$ for $n \neq -1$ and $i < d$, and $H_N^d(\mathrm{gr}_I(R))_n = 0$ for $n \geq 0$, where N denotes the homogeneous maximal ideal of $\mathrm{gr}_I(R)$ ([TI]).

These theorems provide necessary and sufficient conditions for depth $R[It]$ to be at its maximum. However, very little has been written concerning the relationship between depth $R[It]$ and depth $\mathrm{gr}_I(R)$ in more generality. Schenzel touched on this topic in [Sch2] and [Sch3], in which he gave necessary and

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