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CENTROAFFINE IMMERSIONS OF CODIMENSION TWO AND PROJECTIVE HYPERSURFACE THEORY

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Affine differential geometry developed by Blaschke and his school [B] has been reorganized in the last several years as geometry of affine immersions. An immersion f of an n-dimensional manifold M with an affine connection ∇ into an $(n+1)$-dimensional manifold $\tilde{M}$ with an affine connection $\tilde{\nabla}$ is called an affine immersion if there is a transversal vector field ξ such that $\tilde{\nabla}_X f_*(Y) = f_*(\nabla_X Y) + h(X,Y)\xi$ holds for any vector fields X, Y on M^n. When $f: M^n \to \mathbf{R}^{n+1}$ is a nondegenerate hypersurface, there is a uniquely determined transversal vector field ξ, called the affine normal field, an essential starting point in classical affine differential geometry. The new point of view allows us to relax the nondegeneracy condition and gives us more freedom in choosing ξ; what this new viewpoint can accomplish in relating affine differential geometry to Riemannian geometry and projective differential geometry can be seen from [NP1], [NP2], [NS] and others. For the definitions and basic formulas on affine immersions, centroaffine immersions, conormal (or dual) maps, projective flatness, etc., the reader is referred to [NP1]. These notions will be generalized to codimension 2 in this paper.

In this paper we present a systematic study of centroaffine immersions of an n-manifold into $\mathbf{R}^{n+2} - \{0\}$. Such immersions were studied in [W] by adhering to the original features (including apolarity and local convexity assumption) of the Blaschke theory as much as possible. Our approach is more general in that we follow the spirit of the recent development mentioned above. In particular, our work is motivated by, and applied to, projective differential geometry.

The paper is organized as follows. In Section 1 we develop the basic machinery for centroaffine immersions of codimension 2, obtain two fundamental forms h and T and two cubic forms C and δ. The vanishing of T or h is given a geometric

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