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GENERATORS FOR A MAXIMALLY DIFFERENTIAL IDEAL IN POSITIVE CHARACTERISTIC

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Introduction

In this note we give the structure of maximally differential ideals in a Noetherian local ring of prime characteristic $p > 0$, in terms of their generators. More precisely, we prove the following result:

THEOREM 4. *Let A be a Noetherian local ring of prime characteristic $p > 0$ with maximal ideal $\mathfrak{m}$. Let I be a proper ideal of A. Suppose $n = \mathrm{emdim}(A)$ and $r = \mathrm{emdim}(A/I)$. If I is maximally differential under a set of derivations of A then there exists a minimal set $x_1, \ldots, x_n$ of generators of $\mathfrak{m}$ such that $I = (x_1^p, \ldots, x_r^p, x_{r+1}, \ldots, x_n)$.*

This result was proved by the author in [3, Lemma 2.2], under the additional hypothesis that A is complete and I is maximally differential under a set of k-derivations of A, where k is a coefficient field of A.

Using the methods we use to prove the above result we give a different proof for Harper's Theorem (as called by H. Matsumura, [Cf. [4, Theorem on p. 206]]). The following formulation of Harper's Theorem is due to S. Yuan [5]:

"Let A be a differentially simple ring of positive characteristic p. Then A is local. Let $\mathfrak{m}$ be the maximal ideal of A and let $n = \dim_{A/\mathfrak{m}}(\mathfrak{m}/\mathfrak{m}^2)$. If $n < \infty$ then

$$A \cong k[X_1, X_2, \ldots, X_n]/(X_1^p, X_2^p, \ldots, X_n^p),$$

where k is a field and $X_1, X_2, \ldots, X_n$ are indeterminates over k."

Our proof of Harper's Theorem is very straightforward and is much simpler than the original proof by L. Harper [1] and S. Yuan's proof, both of which involve somewhat complicated computations.

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