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ON L-FUNCTIONS ASSOCIATED WITH THE VECTOR SPACE OF BINARY QUADRATIC FORMS

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Introduction

The purpose of this paper is to prove functional equations of L-functions associated with the vector space of binary quadratic forms and determine their poles and residues. For a commutative ring K, let $V(K)$ be the set of all symmetric matrices of degree 2 with coefficients in K. In $V(\mathbf{C})$, we consider the inner product

$$\langle x, y\rangle = \mathrm{tr}(xy^{\iota}) \quad (x, y \in V(\mathbf{C})),$$

where $y^{\iota} = \begin{pmatrix} y_3 & -y_2 \\ -y_2 & y_1 \end{pmatrix}$ for $y = \begin{pmatrix} y_1 & y_2 \\ y_2 & y_3 \end{pmatrix}$. For $i = 1,2$, we set

$$V_i = \{x \in V(\mathbf{R}) \mid (-1)^i \det x > 0\},$$

and $V_i(\mathbf{Q}) = V(\mathbf{Q}) \cap V_i$. We define two subsets of $V_1(\mathbf{Q})$ by

$$V_1'(\mathbf{Q}) = \{x \in V_1(\mathbf{Q}) \mid \sqrt{-\det x} \notin \mathbf{Q}\},$$

$$V_1''(\mathbf{Q}) = \{x \in V_1(\mathbf{Q}) \mid \sqrt{-\det x} \in \mathbf{Q}\}.$$

Let $G = GL_2(\mathbf{C})$, $G(\mathbf{R}) = GL_2(\mathbf{R})$, $G(\mathbf{Q}) = GL_2(\mathbf{Q})$, and $G^+ = \{g \in G(\mathbf{R}) \mid \det g > 0\}$. Then G acts on $V(\mathbf{C})$ by $\rho(g)x = gx{}^tg$ for $x \in V(\mathbf{C})$ and $g \in G$, and the triple (G, ρ, V) is a prehomogeneous vector space with the singular set $S = \{x \in V \mid \det x = 0\}$. For $g = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$, set $dg = (\det g)^{-2}\Pi_{1\le i,j\le 2}\, dg_{ij}$. Then dg defines a Haar measure on G^+. We consider the measures $dx = dx_1dx_2dx_3$ and $\omega(x) = |\det x|^{-3/2}\,dx$ $\left(x = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}\right)$ on $V(\mathbf{R})$. The measure $\omega(x)$ is invariant under the action of G^+. Let Γ be a subgroup of $SL_2(\mathbf{Z})$ of finite index. We assume $\{\pm 1\} \subset \Gamma$. For $x \in V(\mathbf{R})$, let $G_x^+ = \{g \in G^+ \mid \rho(g)x = x\}$,

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