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ON THE STRUCTURE OF LOCAL COHOMOLOGY MODULES FOR MONOMIAL CURVES IN P_K^3

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1. Introduction

Our setting for this paper is projective 3-space P_K^3 over an algebraically closed field K. By a curve $C \subset P_K^3$ is meant a 1-dimensional, equidimensional projective algebraic set, which is locally Cohen-Macaulay. Let $M(C) = \bigoplus_{n \in \mathbf{Z}} H^1(P_K^3, \mathcal{I}_C(n))$ be the Hartshorne-Rao module of finite length (cf. [R]). Here $\mathbf{Z}$ is the set of integers and $\mathcal{I}_C$ the ideal sheaf of C. In [GMV] it is shown that $M(C) \cong H^1_{\underline{m}}(\bar{R})$, where $\bar{R} = R/I(C) = K[x_0, \dots, x_3]/I(C)$, $I(C)$ is the homogeneous ideal of C, $\underline{m} = (x_0, \dots, x_3)R$ and $H^1_{\underline{m}}(M)$ is the first local cohomology module of the R-module M with respect to $\underline{m}$. Thus there exists a smallest nonnegative integer $k \in \mathbf{N}$ such that $\underline{m}^k H^1_{\underline{m}}(\bar{R}) = 0$, (see also the discussion on the 1-st local cohomology module in [GW]). Also in [GMV] it is shown that $k = 0$ if and only if C is arithmetically Cohen-Macaulay and C is arithmetically Buchsbaum if and only if $k \leq 1$. We therefore have the following natural definition.

DEFINITION 1.1. For a curve $C \subseteq P_K^3$, C is said to be strictly k-Buchsbaum if k is minimal in $\mathbf{N}$ such that $\underline{m}^k H^1_{\underline{m}}(\bar{R}) = 0$. C is said to be k-Buchsbaum if $\underline{m}^k H^1_{\underline{m}}(\bar{R}) = 0$.

If C is strictly k-Buchsbaum, then we set $k = k(C)$ and call $k(C)$ the Buchsbaum number of C.

It is our purpose in this paper to investigate for the class of monomial curves $C(n_1, n_2, n_3) \subset P_K^3$ the integer $k(C(n_1, n_2, n_3))$. These curves are defined by their generic zero $(s^{n_3}, s^{n_3-n_1}t^{n_1}, s^{n_3-n_2}t^{n_2}, t^{n_3})$, where $n_1 < n_2 < n_3$ are positive integers and $\text{g.c.d.}(n_1, n_2, n_3) = 1$. For some of these curves $k(C(n_1, n_2, n_3))$ was obtained in [FH], [H] and [FV] and we will discuss some of these results as consequences of out own investigations (see also [HV] and [MM]).

Our own main result is that $k(C) = \text{diam}(M(C))$ for all monomial curves in

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