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INTEGRAL GEOMETRY UNDER CUT LOCI IN COMPACT SYMMETRIC SPACES

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Dedicated to Professor Masaru Takeuchi on his sixtieth birthday

Introduction

The theory of integral geometry has mainly treated identities between integral invariants of submanifolds in Riemannian homogeneous spaces like as $\int_G I(M \cap gN)\, d\mu_G(g)$, where M and N are submanifolds in a Riemannian homogeneous spaces of a Lie group G and $I(M \cap gN)$ is an integral invariant of $M \cap gN$. For example Poincaré's formula is one of typical identities in integral geometry, which is as follows. We denote by $M(\mathbf{R}^2)$ the identity component of the group of isometries of the plane $\mathbf{R}^2$ with a suitable invariant measure $\mu_{M(\mathbf{R}^2)}$. The Poincaré's formula for two curves c_1 and c_2 in R^2 is given by

$$\int_{M(\mathrm{R}^2)} \#(c_1 \cap gc_2)\, d\mu_{M(\mathrm{R}^2)} = 2L(c_1)L(c_2),$$

where $\#(X)$ denotes the number of the points of X and $L(c)$ denotes the length of c. See I.7.2 Poincaré's formula in [15] for more information about it. Chern [3], Kurita [9], Brothers [2] and Howard [7] extended this formula to the case of Riemann homogeneous spaces. We use the notation in Howard [7]. Let M and N be submanifolds of finite volume in a Riemannian homogeneous space G/K of a Lie group G which satisfy $\dim(G/K) \leq \dim M + \dim N$. Then

$$\int_G \mathrm{vol}(M \cap gN)\, d\mu_G(g) = \int_{M \times N} \sigma_K(T_x^\perp M,\, T_y^\perp N)\, d\mu_{M\times N}(x, y).$$

σ_K is an integral invariant, which is defined in Section 1. In the case of $G/K = \mathbf{R}^2$, σ_K is constant and the above formula implies the Poincaré's one. More generally

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