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ON SIEGEL MODULAR FORMS PART II

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1. Introduction

In this paper we compute dimension formulas for rings of Siegel modular forms of genus $g = 3$. Let denote $\Gamma_g(2)$ the main congruence subgroup of level two, $\Gamma_{g,0}(2)$ the Hecke subgroup of level two and Γ_g the full modular group. We give the dimension formulas for genus $g = 3$ for the above mentioned groups Γ and determine the graded ring $A(\Gamma_3(2))$ of modular forms with respect to $\Gamma_3(2)$.

The dimension formula in the case $\Gamma = \Gamma_3$ was first given by Tsuyumine in [T1]. Tsuyumine, following a method of Igusa, used the sequence

$$0 \to \chi_{18}A(\Gamma_3) \to A(\Gamma_3) \to S(2,8)$$

where χ_{18} is a cusp form of weight 18 defining the closure of the hyperelliptic locus and $S(2,8)$ is the graded ring of invariants of binary 8-forms. Tsuyumine uses the structure of $S(2,8)$, given by Shioda [Sh], and restriction of a bigger ring $A'(\Gamma_3)$ with respect to a second divisor.

For our generalization of Tsuyumine's result we use a more direct approach. In [R] we computed the ring of modular forms for $\Gamma_3(2,4)$ (the Igusa subgroup of level two). Principally this allows to compute all rings of modular forms for subgroups Γ with $\Gamma_3(2,4) \subset \Gamma \subset \Gamma_3$. However, this involves subtle computations of rings of invariants with respect to the finite group $\Gamma/\Gamma_3(2,4)$. It turns out that the computation is simplified by constructing a certain central extension H_g of $\Gamma_g/\Gamma_g(2,4)$. This group seems to be of independent interest because of its importance in coding theory. The main ingredient is a decomposition of Bruhat type for the group H_g. This decomposition is closely connected with the theory of partial Fourier transformation.

Finally, in the last chapter we give a characterization of decomposable points in the Satake compactification, which gives another method for computing $A(\Gamma_3(2))$.

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