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INTERSECTION THEORY FOR TWISTED COHOMOLOGIES AND TWISTED RIEMANN'S PERIOD RELATIONS I

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To the memory of Professor Michitake Kita

Introduction

The beta function $B(\alpha, \beta)$ is defined by the following integral

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt,$$

where $\arg t = \arg(1-t) = 0$, $\Re\alpha$, $\Re\beta > 0$, and the gamma function $\Gamma(\alpha)$ by

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt,$$

where $\arg t = 0$, $\Re\alpha > 0$. By the use of the well known formulae

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \quad \Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \Gamma(\alpha)\Gamma(1-\alpha) = \frac{\pi}{\sin \pi\alpha},$$

we get the following formula:

$$B(\alpha, \beta)B(-\alpha, -\beta) = 2\pi i\Big(\frac{1}{\alpha} + \frac{1}{\beta}\Big)\Big(-\frac{\exp(2\pi i(\alpha+\beta)) - 1}{(\exp(2\pi i\alpha) - 1)(\exp(2\pi i\beta) - 1)}\Big).$$

If we regard the interval (0,1) of integration as a twisted cycle defined by the multi-valued function $t^\alpha(1-t)^\beta$, the factor

$$-\frac{\exp(2\pi i(\alpha+\beta)) - 1}{(\exp(2\pi i\alpha) - 1)(\exp(2\pi i\beta) - 1)}$$

is nothing but the twisted self-intersection number ([KY1]) of the cycle (0,1). It is quite natural to think that the factor

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