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SPHERICAL FUNCTIONS ON ORTHOGONAL GROUPS

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Introduction

Let G be a $\mathfrak{p}$-adic connected reductive algebraic group and K a maximal compact subgroup of G. In [4], Casselman obtained the explicit formula of zonal spherical functions on G with respect to K on the assumption that K is special. It is known (Bruhat and Tits [3]) that the affine root system of algebraic group which has good but not special maximal compact subgroup is A_1, C_2, or B_n $(n > 3)$, and all B_n-types can be realized by orthogonal groups. Here the assumption "good" is necessary for the Satake's theory of spherical functions.

Thus in this paper we write down explicitly the zonal spherical functions on all $\mathfrak{p}$-adic (we assume that $\mathfrak{p}$ does not lie over 2 for the convenience of calculation) orthogonal groups but the case of even dimensional split orthogonal groups (this case is contained in the work of Casselman) and determine the image of Satake transform. To do so, we use Macdonald's idea by which he has obtained explicit formula for the $\mathfrak{p}$-adic Chevalley group.

Now we recall briefly some basic notion of Satake transform. Let $\mathbf{L}(G)$ be the set of all compactly supported continuous functions on G with values in $\mathbf{C}$. We put

$$L(G, K) = \{f \in L(G) \mid f(ugu') = f(g) \quad \text{for all} \quad u, u' \in K, g \in G\}.$$

For $f_1, f_2 \in L(G, K)$, we define their product by the convolution

$$(f_1 * f_2)(g) = \int_G f_1(gg_1^{-1}) f_2(g_1) dg_1,$$

where $g \in G$, and dg_1 is the bi-invariant Haar measure on G normalized by the condition that the volume of K is equal to 1. The multiplication gives the structure

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