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ON THE ANALYTIC STRUCTURE OF CERTAIN INFINITE DIMENSIONAL TEICMÜLLER SPACES

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Introduction

It is well known since long time that quasiconformally different finite Riemann surfaces give rise to biholomorphically nonequivalent Teichmüller spaces except for a few obvious cases (cf. [R], [E-K]). This is deduced as an application of Royden's theorem asserting that the Teichmüller metric is equal to the Kobayashi metric. For the case of infinite Riemann surfaces, however, it is still unknown whether or not the corresponding result holds, although it has been shown by F. Gardiner [G] that Royden's theorem is also valid for the infinite dimensional Teichmüller spaces. On the other hand, recent activity of several mathematicians shows that the infinite dimensional Teichmüller spaces are interesting objects of complex analytic geometry (cf. [Kru], [T], [N], [E-K-K]). Therefore, based on the generalized form of Royden's theorem, one might well look for further insight into Teichmüller spaces by studying the above mentioned nonequivalence question.

In the present article, we shall restrict our attention to the connected sum of infinitely many finite Riemann surfaces and show that, if their *necks* are sufficiently long at infinity, one can naturally associate to each quasiconformal equivalence class of such surfaces the cofinal equivalence class of a sequence of nonnegative integers which distinguishes the infinitesimal forms of the Teichmüller metric (cf. Proposition 3.3). For that we shall show in section one, that the space of integrable holomorphic quadratic differentials decomposes asymptotically as the surface is pinched along a simple closed geodesic to two hyperbolic surfaces (cf. Theorem 1.5). The proof of this fact relies on the method of solving the $\bar{\partial}$-equation with L^2 estimates as developed by [A-V], [O-1,2] and [D] on complete Kähler manifolds. This, combined with an elementary argument on the quasi-isometric equivalence of normed vector spaces, allows us to distinguish the infinitesimal forms of the Teichmüller metric by the *direct summands at infinity* of the space of integable

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