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MULTIDIMENSIONAL PROCESS OF ORNSTEIN-UHLENBECK TYPE WITH NONDIAGONALIZABLE MATRIX IN LINEAR DRIFT TERMS

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1. Introduction and results

Let $\mathbf{R}^d$ be the d-dimensional Euclidean space where each point is expressed by a column vector. Let $|x|$ and $\langle x, y\rangle$ denote the norm and the inner product in $\mathbf{R}^d$. Let $Q = (Q_{jk})$ be a real $d \times d$-matrix of which all eigenvalues have positive real parts. Let $\mathbf{X}$ be a process of Ornstein-Uhlenbeck type (OU type process) on $\mathbf{R}^d$ associated with a Lévy process $\{Z_t : t \geq 0\}$ and the matrix Q. Main purpose of this paper is to give a recurrence-transience criterion for the process $\mathbf{X}$ when Q is a Jordan cell matrix and to compare it with the case when Q is diagonalizable. Here by a Lévy process we mean a stochastically continuous process with stationary independent increments, starting at 0. By saying that Q is a Jordan cell matrix (with eigenvalue α) we mean that

$$Q_{jj} = \alpha \text{ for } 1 \leq j \leq d,\ Q_{j,j+1} = 1, \text{ for } 1 \leq j \leq d-1, \text{ and } Q_{jk} = 0 \text{ otherwise.}$$

This paper continues the work [1], where a recurrence-transience criterion is established when Q is diagonalizable. In one dimension the criterion is given by Shiga [5].

Precise definition of the process $\mathbf{X}$ by its infinitesimal generator is given in [1] and [3]. It is a Markov process $(\Omega, \mathscr{F}, \mathscr{F}_t, \mathbf{P}^x, X_t)$ on $\mathbf{R}^d$ such that the process $\{X_t : t \geq 0\}$ under the probability measure $\mathbf{P}^x$ is equivalent to the process $\{\bar{X}_t\}$ defined by

$$\bar{X}_t = e^{-tQ}x + \int_0^t e^{-(t-r)Q} dZ_r, \tag{1.1}$$

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